BrAIn Lab.
Brain and Artificial Intelligence Lab.
SEOULTECH
SEOUL NATIONAL UNIVERSITY OF SCIENCE & TECHNOLOGY

Generative Models: A Probabilistic Perspective

Semin Heo

The Art of Shaping the Data Manifold

2026.01.05

BrainLAB Journal Club
인공지능응용학과 허세민

Overview

Introduction

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The Definition: Distribution Mapping

✓ From Simple to Complex
- Core Concept: 우리가 다루기 쉬운 분포를 복잡한 실제 분포로 변환
- Mathematics: 알려진 분포(Z)에서 샘플링하여, 실제데이터(X)와 유사하게 매핑
  - $Z \sim P_{prior}(z) \rightarrow X \sim P_{data}(x)$

✓ High-dimensional Space
- Sparse Data: 고차원 픽셀 공간에서 의미 있는 데이터는 극히 드묾.
- The Manifold: 실제 데이터는 고차원 공간 속 아주 얇은 막(Manifold) 위에 존재

✓ Intuition: Shaping the Noise (Sculpting)
- Process: 의미 없는 돌덩이를 깎아 예술 작품(Data)을 빚어내는 과정
- Optimization: 생성된 분포 P_{model}가 실제 분포P_{data}와 겹치도록 파라미터(θ) 조정.

Sculpting Probability Distributions from Noise to Reality

Preliminary

02 Known Distributions

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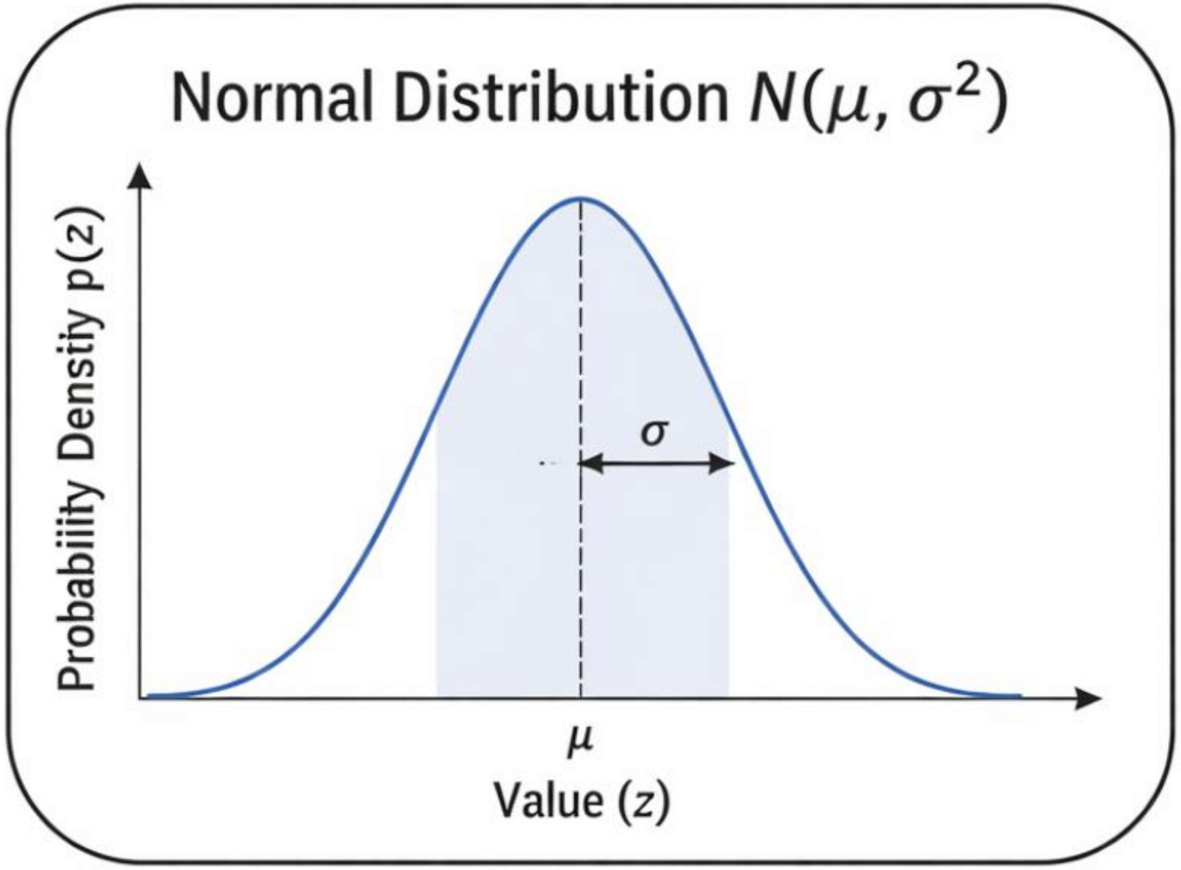
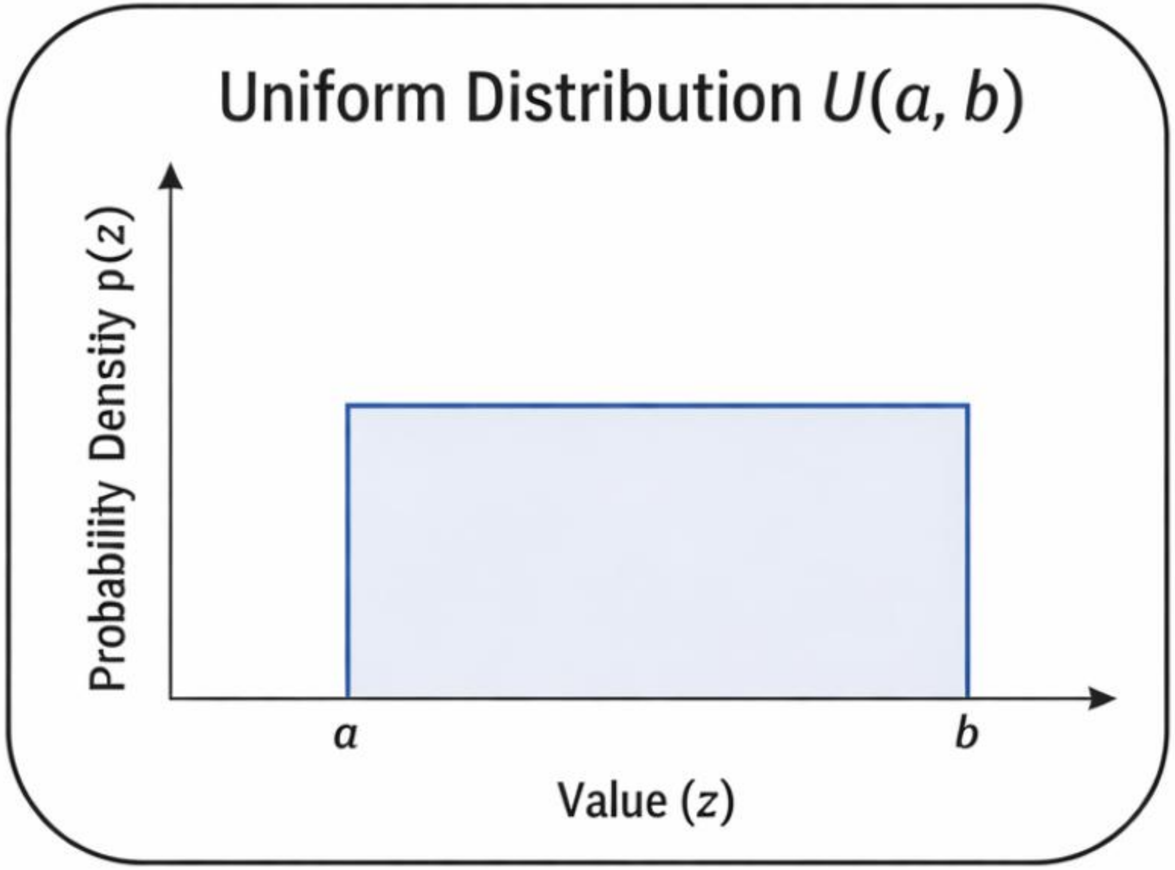
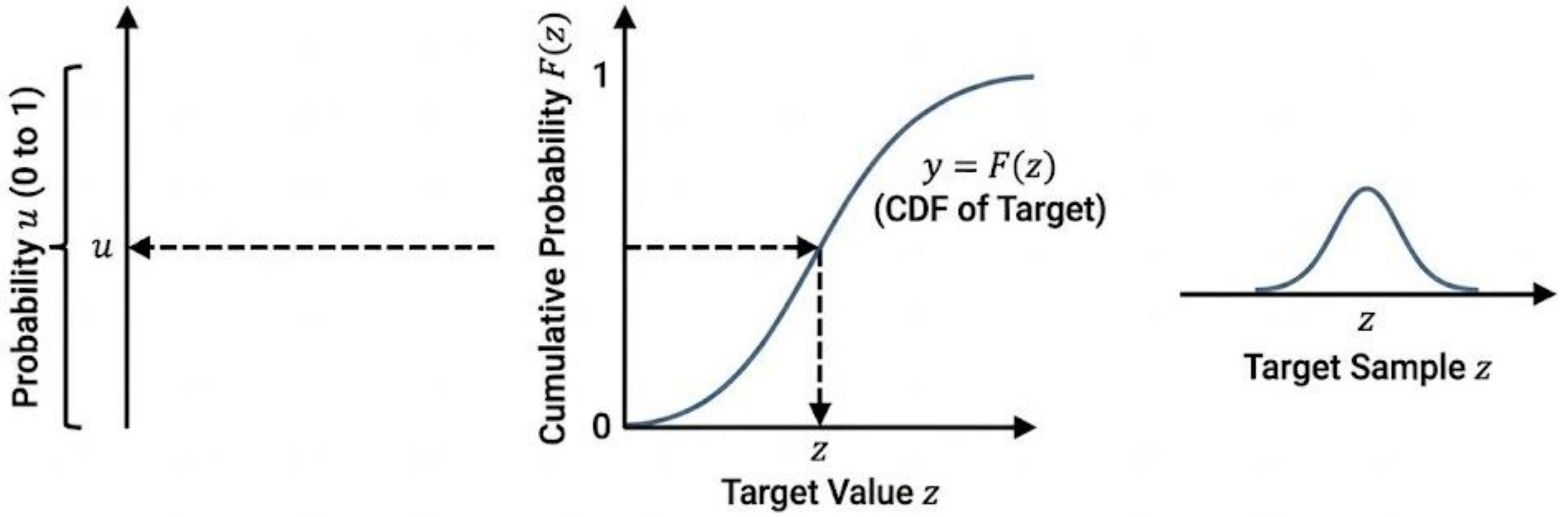




02 Sampling Method

- ✓ **How to Get Z?**
  - 컴퓨터는 진정한 무작위를 모름. 0과 1 사이의 난수(U)만 생성가능.
  - "어떻게 $U \sim \mathrm{Unif}(0,1)$를 $Z \sim \mathrm{N}(0,1)$로 바꿀까?

- ✓ **Cumulative Distribution Function (F)**
  - 특정 값 x까지의 확률을 누적한 함수. (0에서 1사이의 값)
  - **Inverse CDF(F^{-1}): y**축(확률)에서 값을 골라 x축(데이터)을 찾아내는 과정.
    - Mechanism: U를 뽑아서 F^{-1}를 계산하면 우리는 원하는 분포의 샘플 Z가 됨



02 Likelihood

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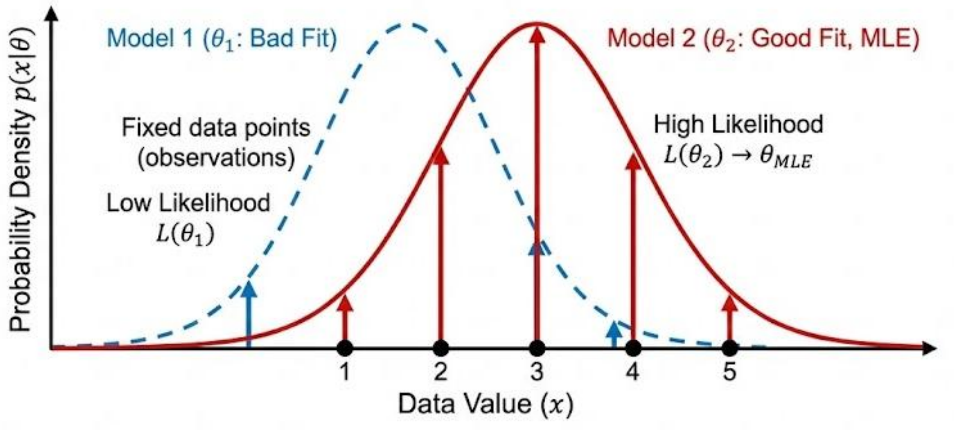
✓ **Score of the Sculptor**
- **Likelihood** ($P(X|\theta)$): 모델 파라미터 θ가 고정되었을 때, 관측된 데이터 X가 등장할 확률.
- **Probability vs Likelihood:**
  - Probability: 분포(θ)가 주어졌을 때 데이터(X)를 예측. (미래)
  - Likelihood: 데이터(X)가 주어졌을 때 분포 (θ)를 평가. (과거/현재)

✓ **Maximum Likelihood Estimation (MLE)**
- "우리가 가진 데이터(X)가 나올 확률을 최대화하는 θ를 찾아라."
- **Log-Likelihood:** 곱셈을 덧셈으로 바꾸어 계산 안정성 확보 ($\sum \log P(x_i|\theta)$)
  - Intuition: "이 모델이 이 데이터를 얼마나 '말이 된다' 고 생각하는지 매기는 점수 "



Taxonomy

03 Models: A taxonomy of Generative Models

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구분	Auto-Regressive (e.g., GPT)	GAN	VAE (Variational AE)	Diffusion (e.g., DDPM)
Mapping & Probability	$x_{<t} \rightarrow x_t$ Explicit Density $P(x) = \prod P(x_t \mid x_{<t})$	$Z \rightarrow X_{fake}$ Implicit Density $x = G(z)$	$X \rightarrow Z \rightarrow X'$ Approximate Density $P(x) \approx \int P(x \mid z)P(z)dz$	$X \leftrightarrow Z$ Diffusion $P(x_0) = \int P(x_t) \prod P(x_{t-1} \mid x_t)$
Objective	Max likelihood	Min-Max Game	Maximize ELBO	Score Matching
Formula	$\prod P(x_t \mid x_{<t})$	$\min_G \max_D V(D, G)$	$E_{x\sim q}[\log P(x \mid z)] - D_{KL}$	$\|\|\epsilon - \epsilon_\theta(x_t, t)\|\|^2$
Intuition	"Chain Rule"	"Forgery Game"	"Manifold Learning"	"Denoising Path"

Details

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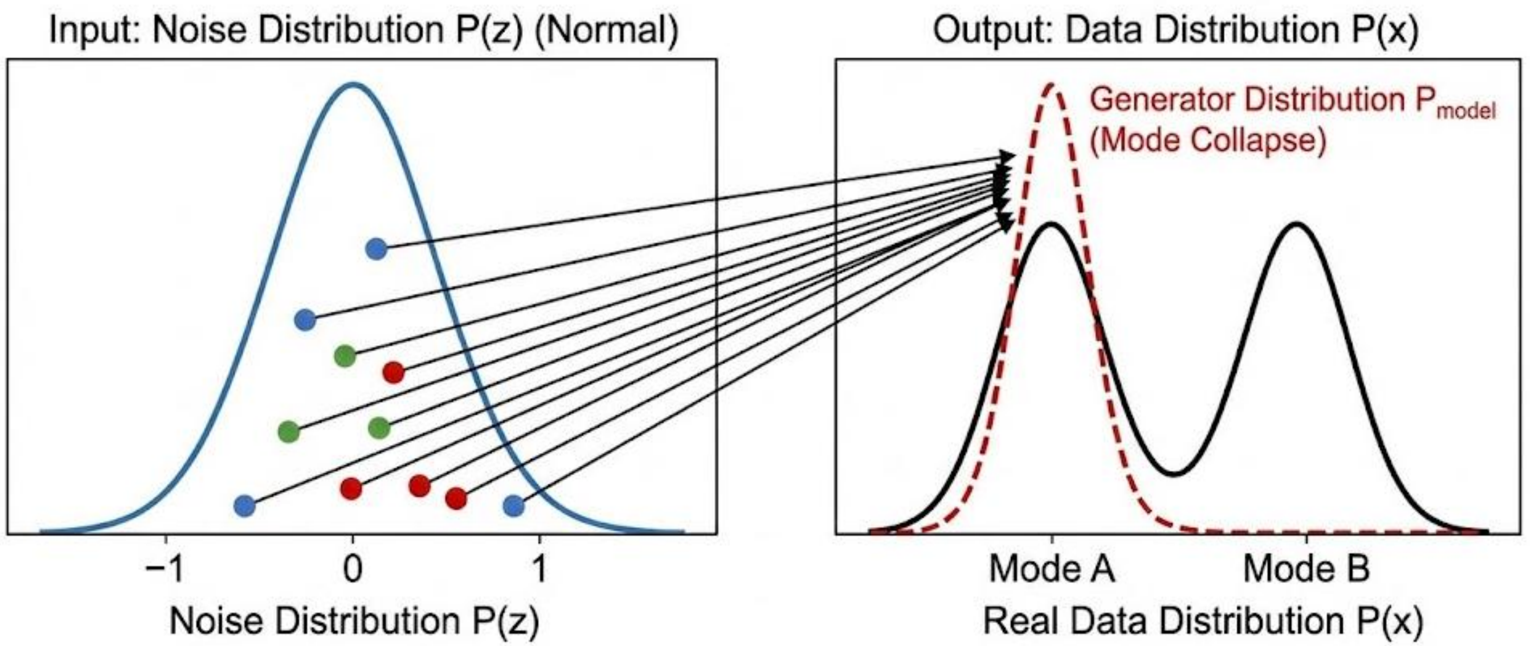
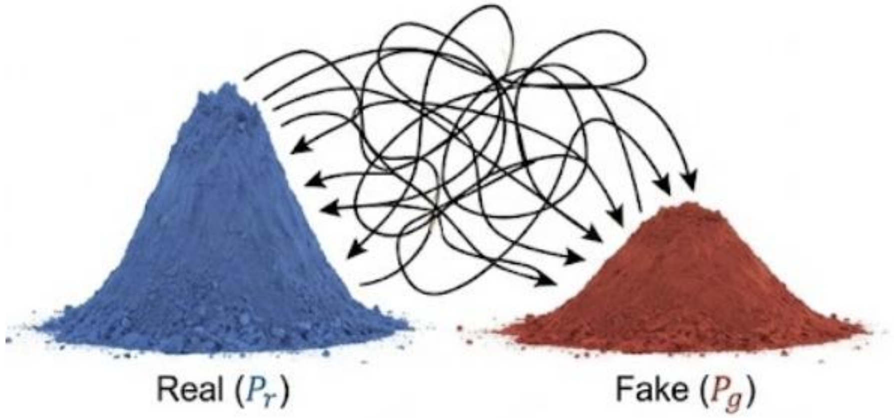
- ✓ **Min-Max Game**
  - **Formula:** $\min_G \max_D V(D,G)$
    - **Discriminator(D):** 진짜(1)와 가짜(0)의 점수 차이를 최대화
    - **Generator(G):** 가짜가 진짜처럼 보여서 차이를 최소화(Min)
- ✓ **Failure Case 1: Mode Collapse** (게으른 생성자)
- ✓ **Failure Case 2: Vanishing Gradient** (너무 강한 판별자)

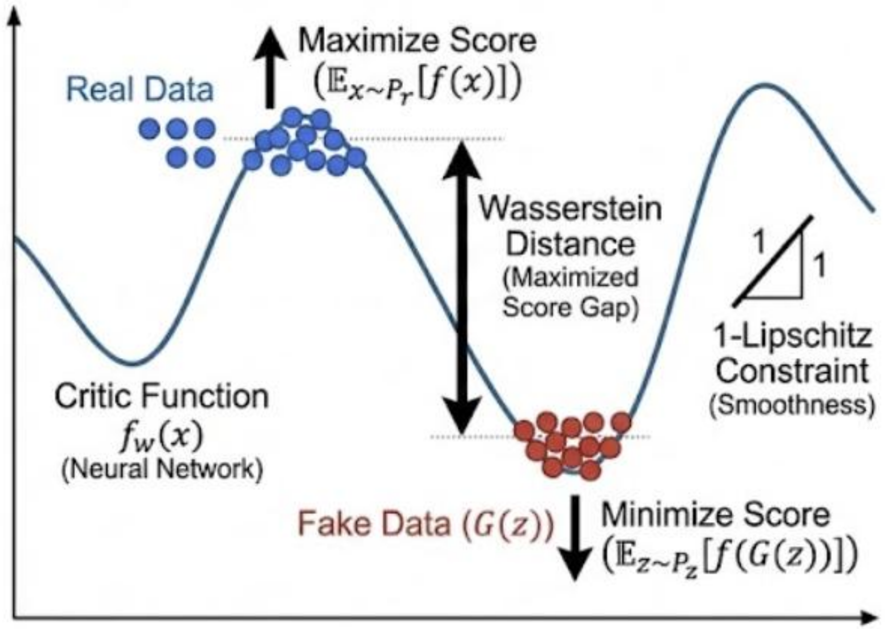


04 WGAN

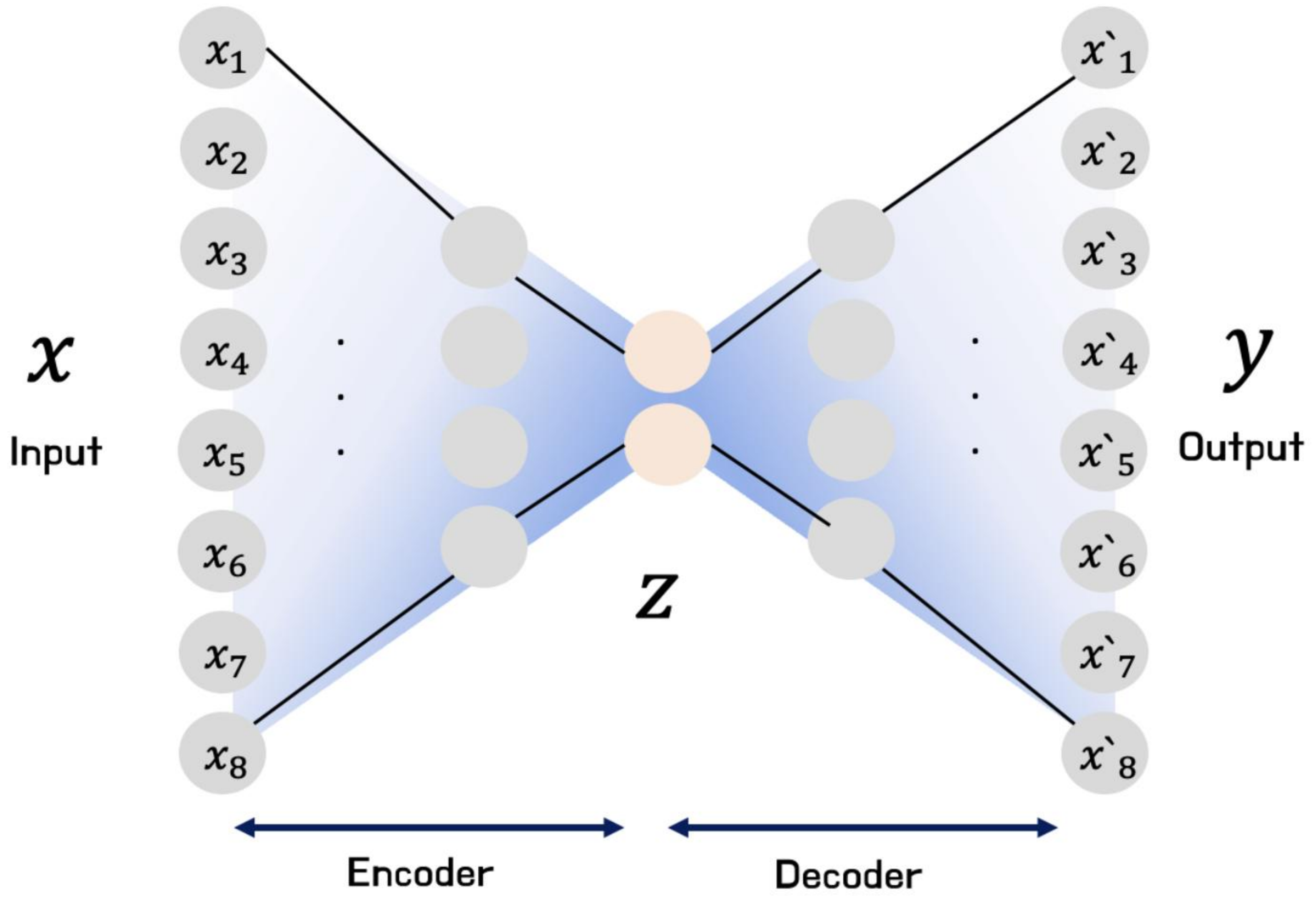
✓ **Original Definition**

- **Primal:** $W(P_r, P_g) = \inf_{\gamma \in \prod(P_r, P_g)} E_{(x,y)\sim\gamma}[||x - y||]$
- **Duality:** $W(P_r, P_g) = \sup_{||f||_L \le 1} (E_{x\sim P_r}[f(x)] - E_{z\sim P(z)}[f(G(z))])$
- **Loss:** $\frac{1}{m}\sum_i f_w(x_i) - \frac{1}{m}\sum_i f_w(G(z_i))$





04 Auto-Encoder



04 Variational Autoencoder

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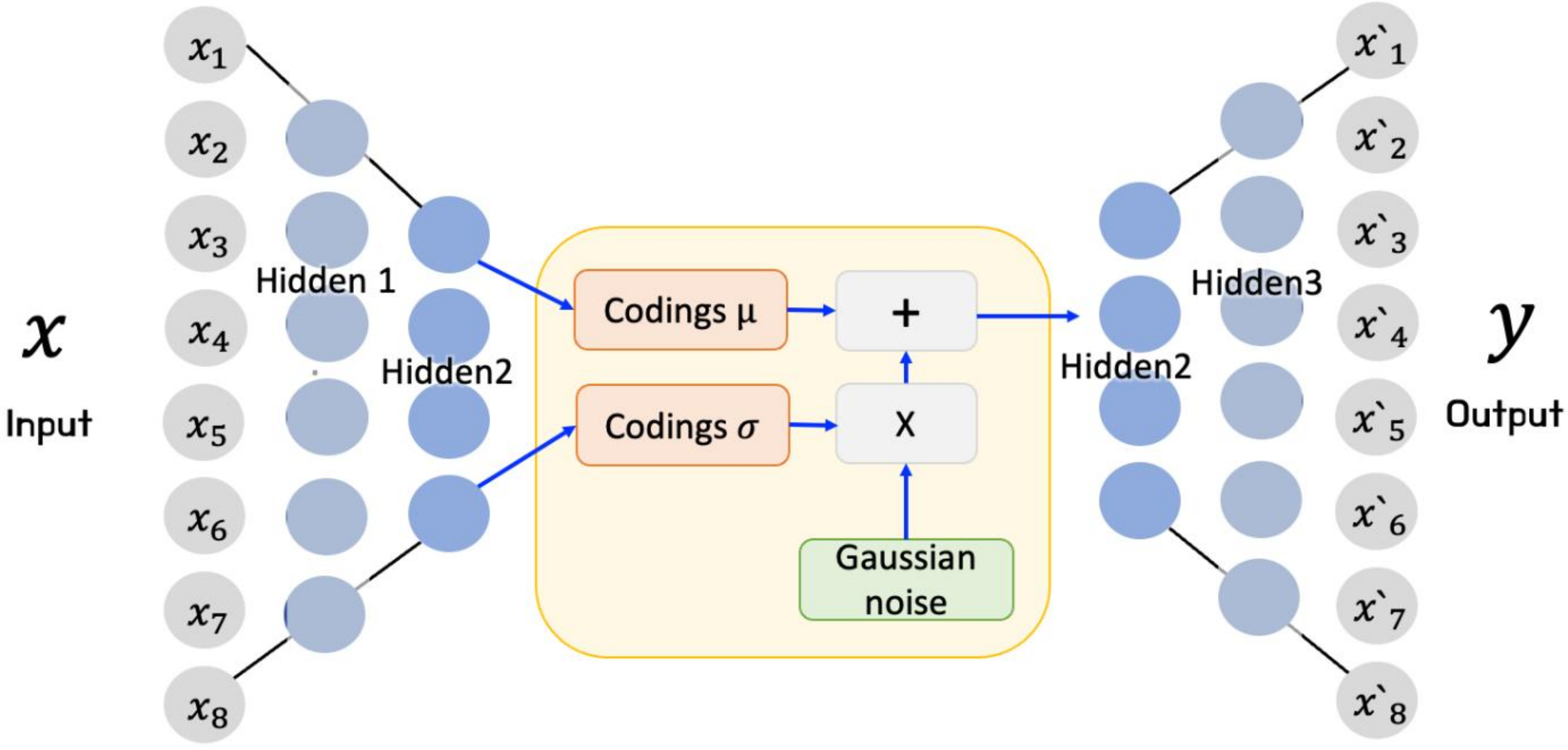

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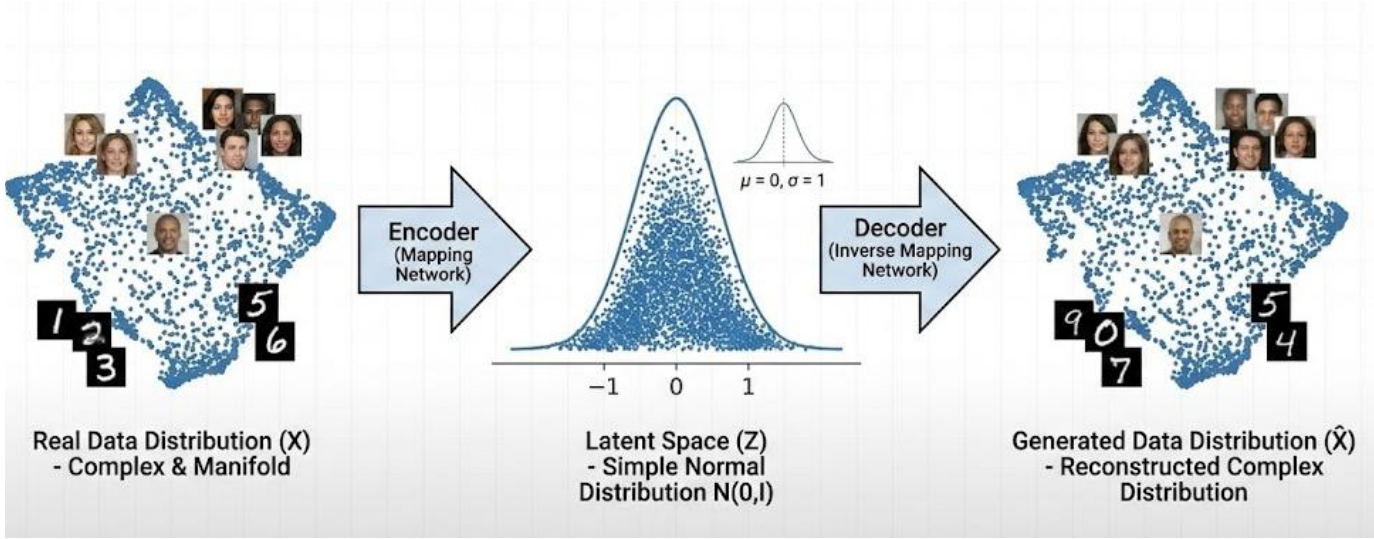
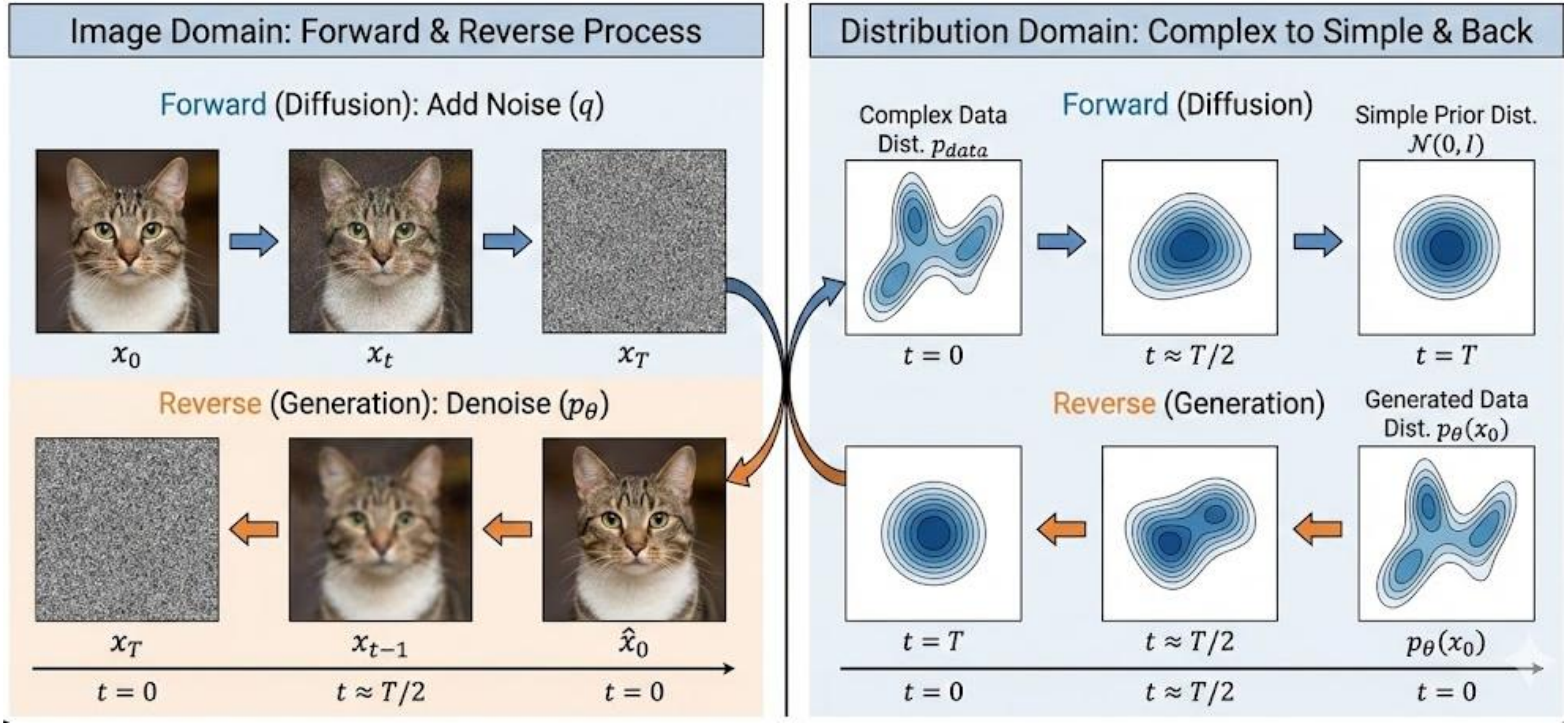
✓ ELBO

- Maximize Likelihood ($\log P(x)$)
- **Intractable Integral:** $P(x) = \int P(x|z)P(z)dz$
- $\log P(x) \geq E_{z\sim Q}[\log P(x|z)] - D_{KL}(Q(z|x)||P(z))$



$$
\begin{aligned}
&\ln p(x) \\
&= \ln \int_z p(x, z) \\
&= \ln \int_z p(x, z)\frac{q(z|x)}{q(z|x)} \\
&\geq \mathbb{E}_{q(z|x)}[\ln \frac{p(x, z)}{q(z|x)}] \\
&= \mathbb{E}_{q(z|x)}[\ln \frac{p(x|z)p(z)}{q(z|x)}] \\
&= \mathbb{E}_{q(z|x)}[\ln p(x|z)] + \mathbb{E}_{q(z|x)}[\ln \frac{p(z)}{q(z|x)}] \\
&= \mathbb{E}_{q(z|x)}[\ln p(x|z)] + \int_z q(z|x) \ln \frac{p(z)}{q(z|x)} \\
&= \mathbb{E}_{q(z|x)}[\ln p(x|z)] - D_{KL}[q(z|x)||p(z)] \\
&= likelihood - KL
\end{aligned}
$$

04 DDPM (ScoreMatching)



04 Flow Matching

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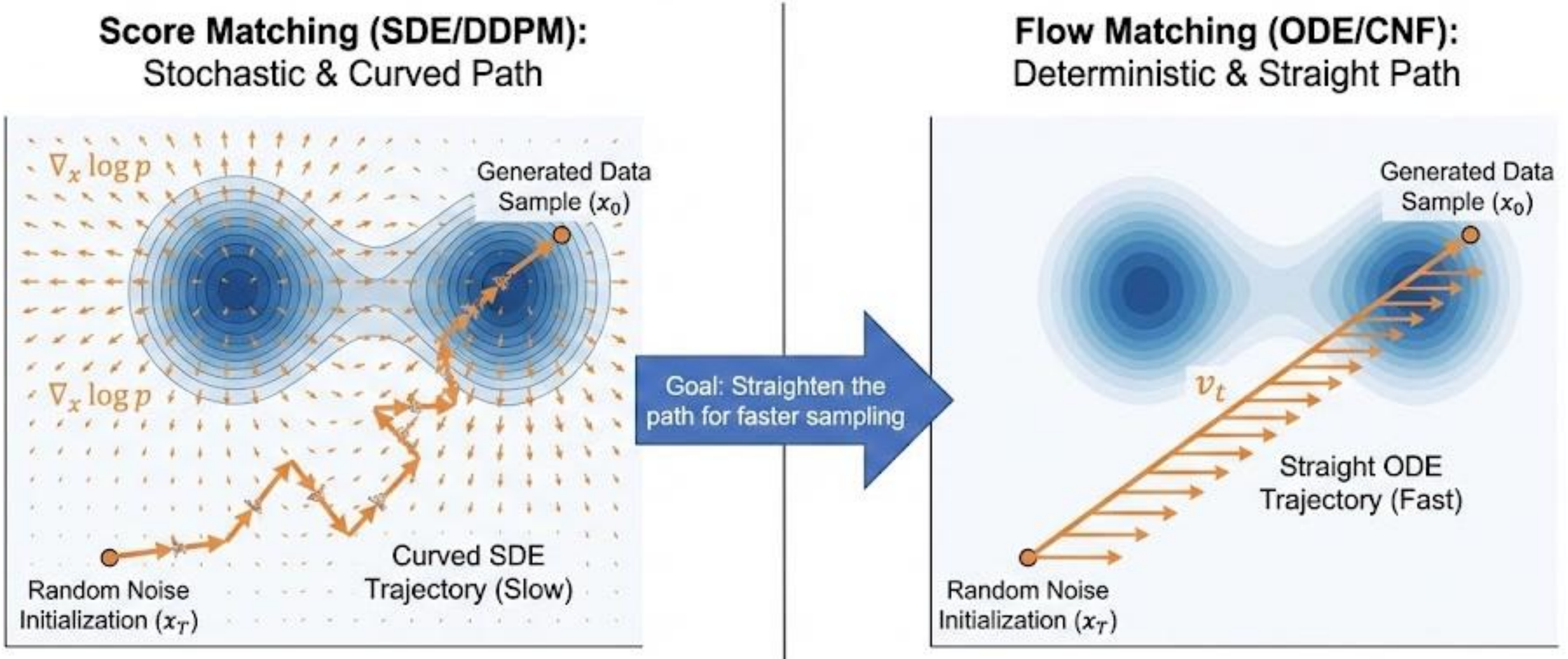

Mechanism: Follows gradients (Score) with added noise. Path is stochastic and inefficient.

Mechanism: Learns the velocity of the straightest path (Optimal Transport). Path is deterministic and efficient.

Conclusions

05 Conclusion

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"Generative Models do not just 'create' data.
They 'understand' the structure of the world"

Thank You

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